

Sensors—Linearity (or lack thereof)

In most situations it is ideal if. . . the digital representation of an analog physical quantity is linearly related to the analog quantity.

Let f(x(t)) represent the digital value of the analog physical quantity x(t)here t is the independent variable, often time, but it can be distance or anything else, at least in theory. Ideally we usually desire

 $f\bigl(x(t)\bigr)=ax(t)$

where *a* is a constant scalar of proportionality. This is a *linear* relationship using the conventional definition of "linearity iff superposition applies." Sensors—Linearity (or lack thereof)

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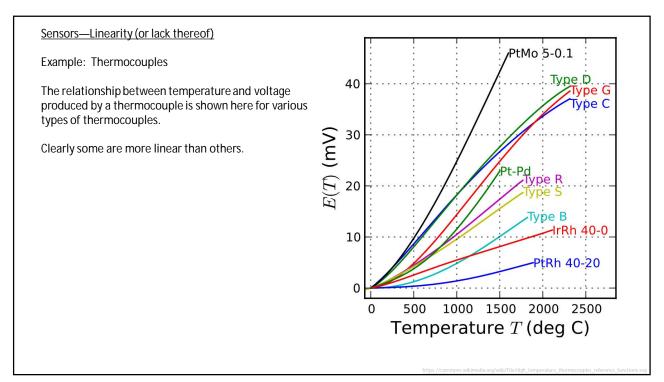
Often the digital version of the signal will offset by some constant value, a bias.

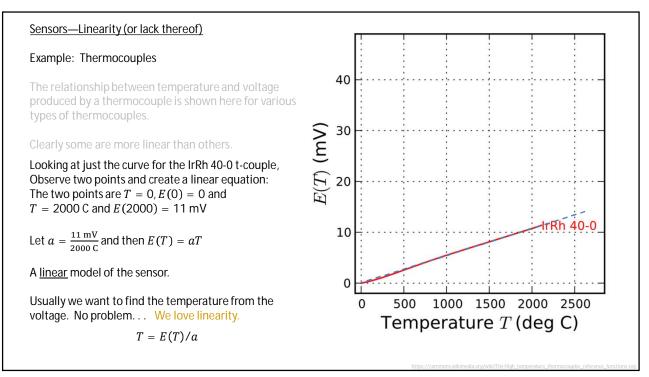
$$f(x(t)) = ax(t) + b$$

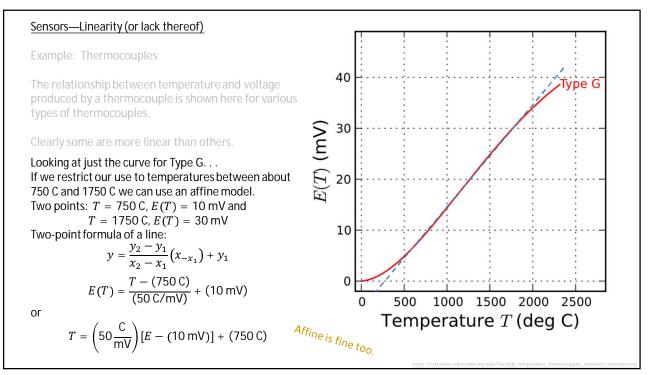
This may appear to be a linear equation (by a high-school definition saying that this very equation is the definition)! But the "+ b" portion of the equation causes superposition to fail. This this is technically not a linear relationship. This equation is mathematically known as an *affine function*.

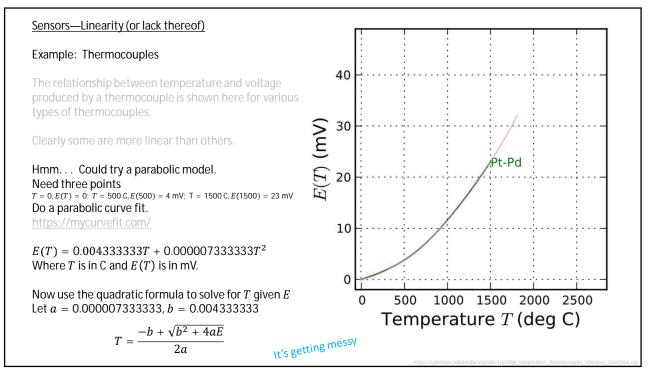
Every linear function is also an affine function (with b = 0).

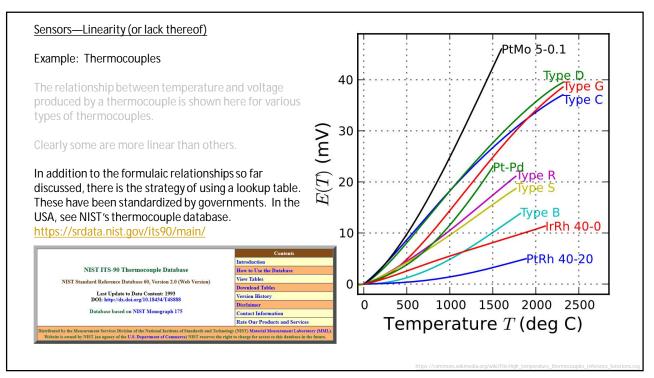
However, since the bias is constant, it can usually be accounted for in software, giving affine relationships almost equal usefulness as linear equations. (Hence perhaps the lack of distinction of this matter at the high-school level.)

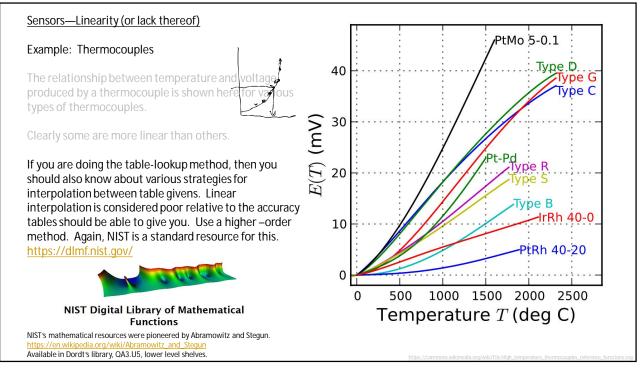




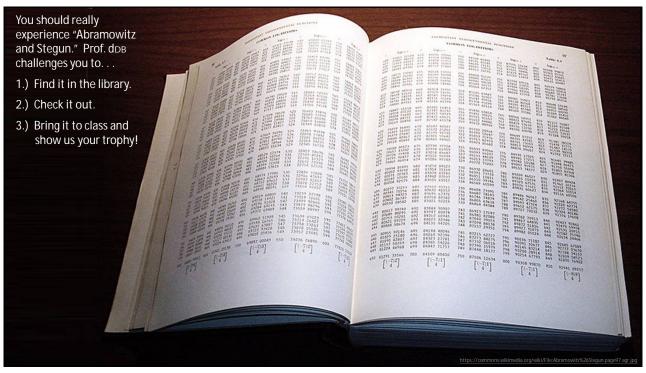


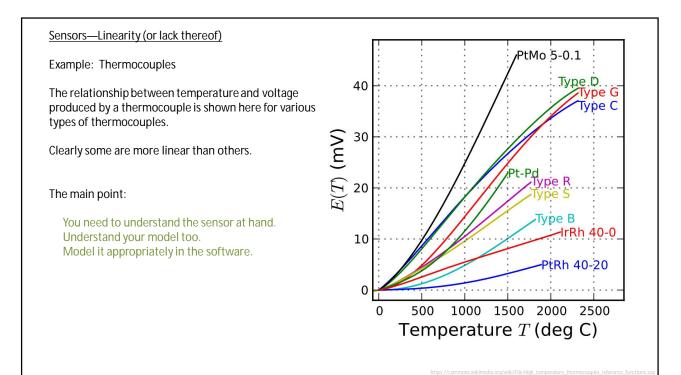












Sidebar item: Decibels

Coming up—characteristics of analog sensors. In some contexts the units of *decibels* are used with analog quantities.

Def'n: A *bel* is the base-ten log of the ratio of two amounts of power.

Example

Cell phone speaker: 0.1 W of power when streaming loud music. Rock concert: 100 kW of power to the loudspeakers. (need a generator on an 18-wheeled trailer.)

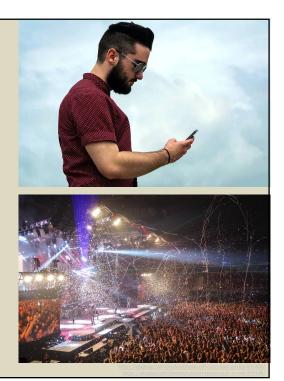
$$\log_{10}\left(\frac{100000}{0.1}\right) = \log(1000000) = 6 B$$

The rock concert requires 6 bels more power.

Def'n: A decibel is 1/10 of a bel.

$$10\log_{10}\left(\frac{100000}{0.1}\right) = 60 \, \mathrm{dB}$$

The rock concert requires 60 dB more power.



Sidebar item: Decibels

Decibel represents a *power* ratio.

If the units are watts, BTU/hr, horsepower, (any unit of *power*) and numerator and denominator have the same units, then. . .

Ratio in dB =
$$10 \log_{10} \left(\frac{P_1}{P_{ref}} \right)$$

But very often the units are not watts!

Say the units are volts.

We know that P = VI and that I = V/R thus $P = V^2/R$

Put that in the decibel formula!

Ratio in dB =
$$10 \log_{10} \left[\frac{(V_1^2)/R}{(V_{ref}^2/R)} \right] = 20 \log \left(\frac{V_1}{V_{ref}} \right)$$

This works with practically all signal units that are NOT POWER.

Thus, if units are power, use $10 \log_{10}(power ratio)$

If units are not power, use 20log_10(not a power ratio)

